

GENERAL

Typos in the original midterm which were corrected during the test have been corrected both in the posted exam and its posted solution.

PART C – INDUCTION

Explanation of grading scheme

This question was graded both “positively” in green and “negatively” in red. The colour of your total grade indicates whether the scheme applied was the positive one or negative one.

The default grading scheme is a negative one: taking marks off for missing central elements in the proof. The majority of the exams were graded this way, and therefore many exams will only have red annotations.

In cases where most of the proof was missing, a positive scheme was used instead: giving grades for central elements of the proof that were found. In this case, there may be red annotations on the exam in addition to the green ones because it was determined during grading that the positive scheme would yield better grades and the negative scheme was abandoned in favour of a positive scheme. When that is the case and the total grade for this question is in green, you can just ignore any red annotations. They are incomplete and irrelevant.

General Comments on Solutions

- In proofs by induction it is standard to define $P(n)$ as a **predicate** function of n , i.e. a function of n that either **has a value of true or false**, so that what needs to be proved is that $\forall n \in D, P(n)$, where D is a subset of \mathbb{N} . Defining $P(n)$ as a function of n that is not a predicate function, for example as the LHS or RHS of an equation, invariably leads to problems in the proof and should be avoided.
- The property that needed to be proved was “for all positive integers”, i.e. for all integers ≥ 1 . Therefore the base case was $n=1$ and not $n=2$. If you have questions about how this is accomplished, please check the solution. The important property to keep in mind is that for any function $f(i)$, $\sum_{i=a}^b f(i) = 0$ whenever $a > b$.
- Finally, **proofs should not be argued from the conclusion**. In other words, you should not start from the conclusion, do stuff to it, and then say “hey, this is all good; I’m done”. You may do this in your rough work to help you figure out what are the elements of the proof, but this is not a proof, just like a list in bullet-point form is not an essay, words put together without articles or verbs are not sentences, and design diagrams are not full programs. All of these are examples of useful, and possibly important, rough work, but they are not finished work.